High-Frequency Behavior of the Bipolar Junction Transistor

Matthew Beckler
beck0778@umn.edu
EE3101 Lab Section 008

December 13, 2006

Abstract

The Hybrid-π model for the small-signal operation of the BJT is a very common model, incorporating a number of extra components from the standard model. In this experiment, we will experimentally determine many of the values of the model's components. By exploiting the frequency response of the Hybrid-π model, we can derive a series of equations to determine the values of these components. We find the values of C_μ, C_π, and r_π to be reasonable, when compared to the manufacturer specified sub-component values. We also compare the frequency bandwidth and transfer function of a couple different amplifier arrangements, including the common-emitter amplifier, the common-emitter/common-base (cascode) amplifier, and a common-collector/common-base amplifier.

1 Introduction

There are two primary concerns with this laboratory experiment. First, we will investigate the accuracy and merit of the Hybrid-π model of the BTJ. This will involve doing experiments to enable us to calculate the values of the internal resistances and capacitances. Second, we will investigate the varying frequency-based metrics of amplifier performance, namely the mid-band gain and the frequency bandwidth. The mid-band gain is classified as an amplifier's gain, when used with signals having frequencies in the middle of the amplifier's band-pass region. The frequency bandwidth is the width (in Hertz) of the amplifier's pass band.

2 Experiments

2.1 Hybrid-π Model

We have not used the Hybrid-π model as it is normally represented. We are ignoring r_μ, which would normally be placed in parallel with C_μ. We will also be ignoring r_o, as we will be shunting that resistance by a much smaller resistance, rendering it ineffective. The revised Hybrid-π model is reproduced below, along with the amplifier circuit we will be using to make a series of measurements to determine the values of C_μ, C_π, and r_π.

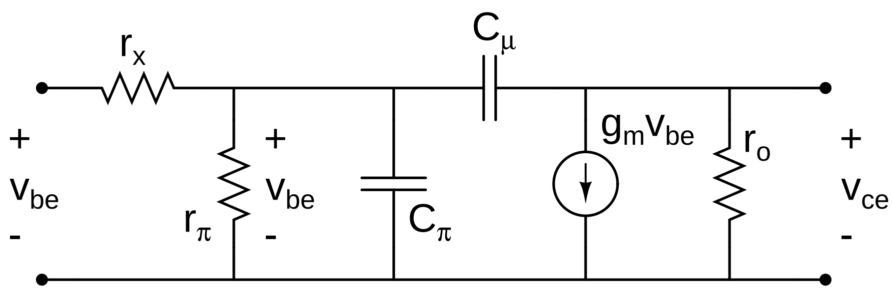

Figure 1: Hybrid-π BJT Model

Under DC operation, the two discrete capacitors will become open circuits, effectively cutting-out the voltage supply. Before we construct this circuit, we need to calculate a few component values, based on our biasing circuit. We have been told some of the values to use, while we must calculate the other values. The given values are listed in the following table, and we will go through the work of calculating the unknown values.

Given Values for Circuit Components

R_s	10 kΩ
V_B	7.5 v
C_s	10 μF
C_e	10 μF
V_{cc}	15 v
I_c	1 mA
β	150
R_L	100 Ω

First, we observe that V_B has been specified at 7.5 v. This sets V_E at 6.8 v. Ad DC, the voltage across R_e is this 6.8 v. The current through this resistor

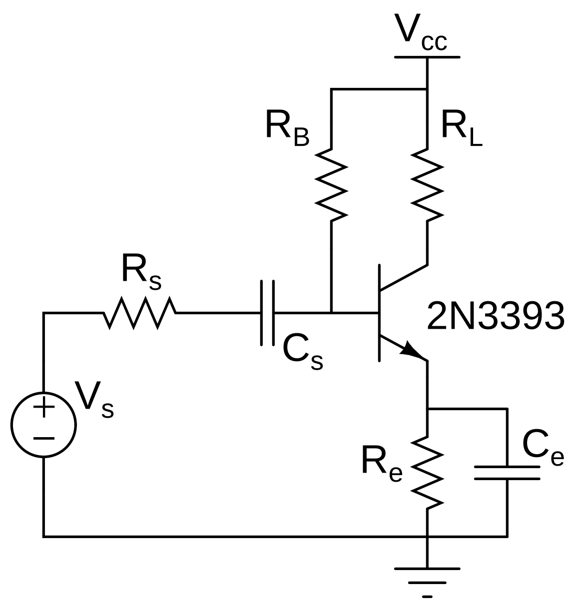

Figure 2: Amplifier Circuit Schematic

is the very nearly the same current that flows through the BJT's collector (neglecting the base-current). We can use the 6.8 v drop across R_e, along with the resistor's current of 1 mA, to calculate the value of R_e:

$$R_e = \frac{6.8v}{1mA} = 6.8k\Omega \approx 22k\Omega||10k\Omega = 6.875k\Omega$$

To find the value of R_B, we observe that at DC, I_B is the current through the base resistor, R_B. Also, by the basic BJT DC-model, $I_C = \beta I_B$. We know that $\beta \approx 150$, and $I_C = 1mA$. We use this to find the base current:

$$I_C = \beta I_B \Rightarrow I_B = \frac{1}{\beta} I_C = \frac{1}{150} \cdot 1mA = 6.66\mu A$$

We know that V_B has been specified to be 7.5 v, leaving a 7.5 v drop across R_B. Having just calculated the current through R_B to be 6.66 μA, we can calculate the resistance value for R_B:

$$R_B = \frac{7.5}{6.66\mu A} \approx 1.125M\Omega$$

Having finished calculating the unknown component values, we can move on to construction and analysis of this circuit. We are interested in measuring the input resistance of the transistor at low frequencies. We use the test voltage suppy, connected to R_s, will behave like a current source. We want to measure the ratio of the input voltage to the input current, which will give us the input resistance. We use a sinusoidal input signal, at 1 kHz. This relatively low frequency effectively shorts-out the discrete capacitors present in the circuit. To measure the input current, we can measure the difference across R_s, and calculate the current. We used the oscilloscope's two probes to measure the voltage on both sides of R_s. We then used the scope's mathematical operations to calculate the difference between the two voltages. This produced a value of approximately 131 mV. We can then calculate the input current:

$$I_s = \frac{131mV}{10k\Omega} = 13.1\mu A$$

Again using the scope to measure from the ground to the transistor's base, we determine the input voltage to be approximately 105 mV. We can then calculate the input resistance of this amplifier:

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{105m}{13.1\mu} = 8.09k\Omega$$

This value seems higher than expected, but matches with what others in the lab section have been getting. This discrepancy from the expected behavior can probably attributed to both parasistic resistances and capacitances in the laboratory equipment, which creates a lot of noise in the oscilloscope's signals. All this extra noise makes it harder to accurately measure peak-to-peak values, especially for automated measurements made with the oscilloscope.

2.2 Frequency Response Part I.

In the next three sections, we will be finding the frequency of the BTJ's single pole, by measuring the frequency response, and finding the -3 dB frequency. The current gain of the BJT behaves according to the following equation:

$$f_H = \frac{1}{2\pi \cdot r_\pi (C_\pi + (1 + g_m R_L) \cdot C_\mu)}$$

We can find f_H by measurement of the -3 dB point, we can calculate g_m from the bias conditions, and R_L is known. This leaves three unknown variables in the previous equation. By modifying circuit conditions and connections, we can get three equations in three unknowns. First, here in section 2, we measure the small signal collector current. We do this by first measuring the difference in voltage across R_s, which allows us to calculate I_{in}. To calculate I_o, we measure the voltage across R_L and then divide by the resistance.

Frequency Response of Initial Amplifier

Frequency (Hz)	$I_{in}(mA)$	$I_o(mA)$	A_I
1 k	0.010	1.910	191
100 k	0.012	1.750	149.57
200 k	0.013	1.780	139.06
300 k	0.014	1.770	127.34
400 k	0.015	1.770	122.07
500 k	0.015	1.770	118.08
600 k	0.015	1.770	116.45
700 k	0.015	1.770	115.69
800 k	0.015	1.750	113.64
900 k	0.017	1.880	111.9
1000 k	0.017	1.830	108.93

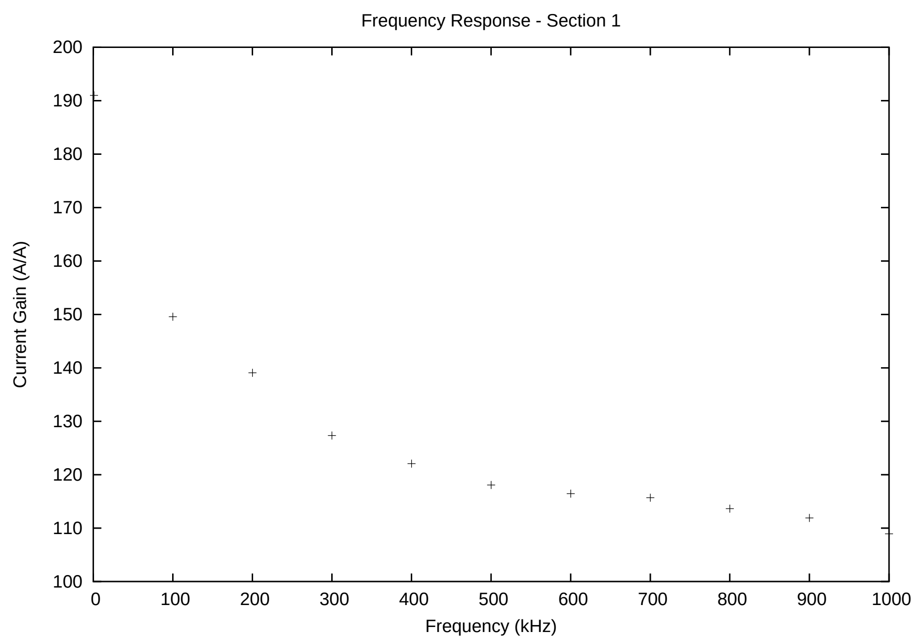

Figure 3: Frequency Response

We calculate the value of g_m:

$$g_m = \frac{i_c}{V_{BE}} = \frac{0.737m}{32.2m} = 23mS$$

To find the corner frequency, we find the frequency where the current gain is 70.7% of the maximum A_I:

$$0.707 \cdot 72 = 50.9$$

This occurs at apprximately 400 kHz, which we can plug back into our equation for f_H:

$$400kHz = \frac{1}{2\pi \cdot r_\pi(C_\pi + 3.3 \cdot C_\mu)}$$

This is the first of three equations we will need to find all three component values.

2.3 Adding a 1 nF Capacitor

Here, we have added a 1 nF capacitor across the existing C_μ, which lies across the Base-Collector junction. Our revised circuit schematic is printed below. The values of i_c and V_{BE} remain the same, meaning that the value of g_m has not changed, and is still 22.8 mS. Again, we have measured the voltage across the input resistor, as well as the voltage across R_L, and calculated the appropriate currents. The data is summarized in the following data table and plot.

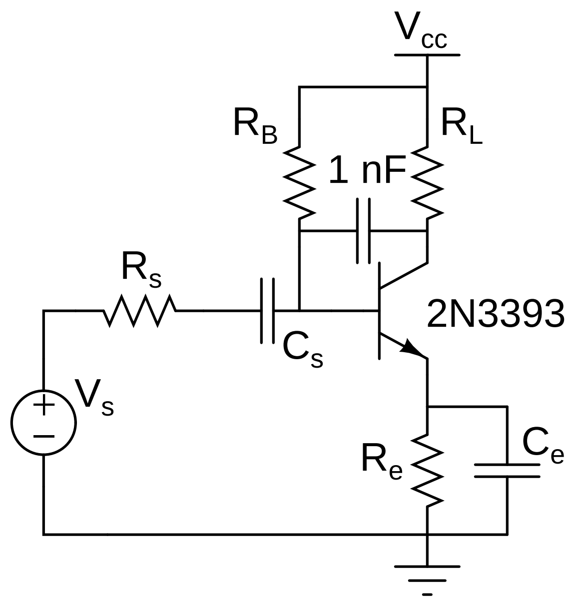

Figure 4: Amplifier Circuit Schematic - Section 3

Frequency Response of Modified Amplifier

Frequency (Hz)	$I_{in}(\mu A)$	$I_o(\mu A)$	A_I
1 k	5.19	750	144.51
5 k	5.69	750	131.81
10 k	5.59	631	112.88
15 k	5.97	531	88.94
20 k	5.97	419	70.18
25 k	5.07	331	65.29
30 k	4.75	331	69.68
35 k	4.47	300	67.11
40 k	4.28	281	65.65
45 k	4.28	219	51.17
50 k	4.88	206	42.21

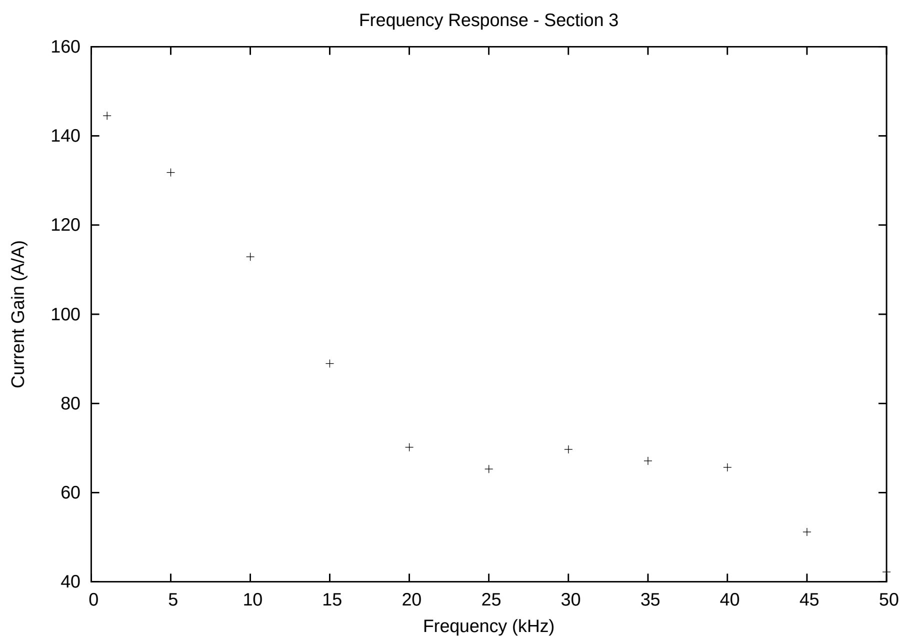

Figure 5: Frequency Response - Section 3

Again, we need to find the -3 dB point. We observe the maximum current gain is 144 (A/A), leading to a -3 dB value of 101 (A/A). This happens at a frequency of approximately 7 kHz. Modifying our equation again, we get:

$$7kHz = \frac{1}{2\pi \cdot r_\pi(C_\pi + 3.3 \cdot (C_\mu + 1nF))}$$

2.4 Increase R_L

In this section, we find the final equation needed. We remove the 1 nF capacitor added in the previous section, and instead we increase the value of R_L by a factor of ten, to a value of $1k\Omega$. We performed our measurement in the exact same way as the previous two sections. Our frequency response data is summarized in the following table and chart.

Frequency Response of Modified Amplifier

Frequency (Hz)	$I_{in}(\mu A)$	$I_o(\mu A)$	A_I
1 k	3.34	780	233.53
5 k	2.97	819	275.76
10 k	2.84	800	281.69
20 k	3.62	763	210.77
30 k	3.75	755	201.33
40 k	3.72	740	198.92
50 k	4.22	720	170.62
60 k	4.13	700	169.49
70 k	4.81	680	141.37
100 k	5.31	609	114.69

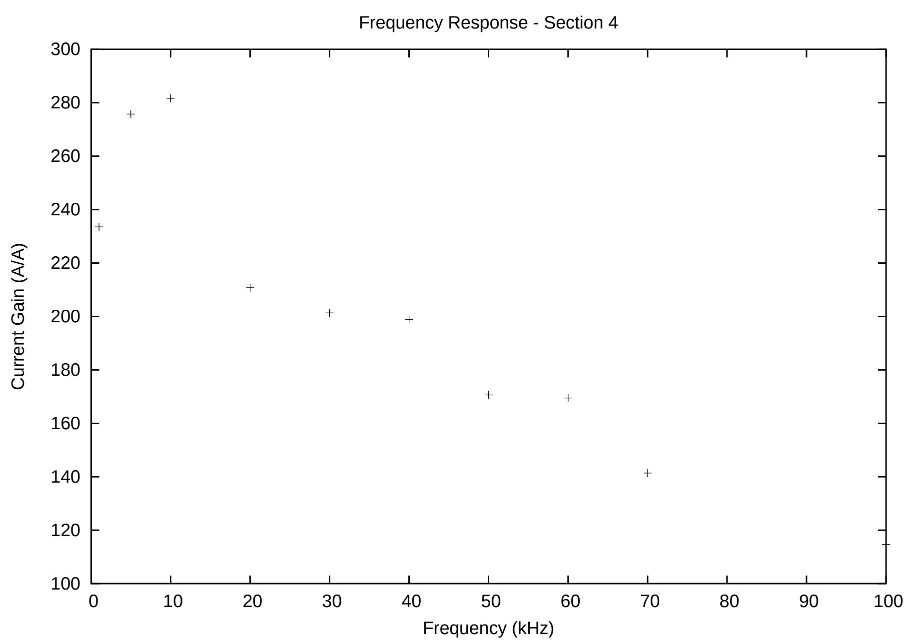

Figure 6: Frequency Response - Section 4

Again, we need to find the -3 dB point. We observe the maximum current gain is 281.69 (A/A), leading to a -3 dB value of approximately 200 (A/A). This happens at a frequency of approximately 35 kHz. Modifying our equation again, we get:

$$35kHz = \frac{1}{2\pi \cdot r_\pi(C_\pi + 33 \cdot C_\mu)}$$

2.5 Calculation of C_μ, C_π, and r_π

In summary, here are the three equations we have just found:

$$\begin{aligned}
400kHz &= \frac{1}{2\pi \cdot r_\pi(C_\pi + 3.3 \cdot C_\mu)} \\
7kHz &= \frac{1}{2\pi \cdot r_\pi(C_\pi + 3.3 \cdot (C_\mu + 1nF))} \\
35kHz &= \frac{1}{2\pi \cdot r_\pi(C_\pi + 33 \cdot C_\mu)}
\end{aligned}$$

At this point, we need to solve these three equations in three unknowns. First, we solve all three for r_π:

$$\begin{aligned}
r_\pi &= \frac{1}{2\pi \cdot 400k \cdot (C_\pi + 3.3 \cdot C_\mu)} \\
r_\pi &= \frac{1}{2\pi \cdot 7k \cdot (C_\pi + 3.3 \cdot (C_\mu + 1nF))} \\
r_\pi &= \frac{1}{2\pi \cdot 35k \cdot (C_\pi + 33 \cdot C_\mu)}
\end{aligned}$$

We set two equal to each other, and get a relation between C_π and C_μ:

$$\begin{aligned}
\frac{1}{2\pi \cdot 400k \cdot (C_\pi + 3.3 \cdot C_\mu)} &= \frac{1}{2\pi \cdot 35k \cdot (C_\pi + 33 \cdot C_\mu)} \\
400k \cdot (C_\pi + 3.3 \cdot C_\mu) &= 35k \cdot (C_\pi + 33 \cdot C_\mu) \\
C_\pi &= 0.452 \cdot C_\mu
\end{aligned}$$

We can substitute this relation back into the other original equation, and one of the equations we've already used:

$$\begin{aligned}
\frac{1}{2\pi \cdot 400k \cdot (C_\pi + 3.3 \cdot C_\mu)} &= \frac{1}{2\pi \cdot 7k \cdot (C_\pi + 3.3 \cdot (C_\mu + 1nF))} \\
400k \cdot (C_\pi + 3.3 \cdot C_\mu) &= 7k \cdot (C_\pi + 3.3 \cdot (C_\mu + 1nF)) \\
400k \cdot ((0.452 \cdot C_\mu) + 3.3 \cdot C_\mu) &= 7k \cdot ((0.452 \cdot C_\mu) + 3.3 \cdot (C_\mu + 1nF)) \\
C_\mu &= 15.67pF \\
C_\pi &= 7.08pF
\end{aligned}$$

Having found the values for the two capacitances, we can now solve for the last unknown value, the resistance r_π:

$$r_\pi = \frac{1}{2\pi \cdot 400k \cdot (7.08pF + 3.3 \cdot 15.67pF)} = 6.77\Omega$$

These three values are all fairly reasonable for the transistors we were working with. Obviously, noisy equipment complicates the very accurate measurements needed, but this method works pretty well.

We also need to calculate the value of r_x for the transistor. We use the following section of the modified Hybrid-π model to determine the value of r_x:

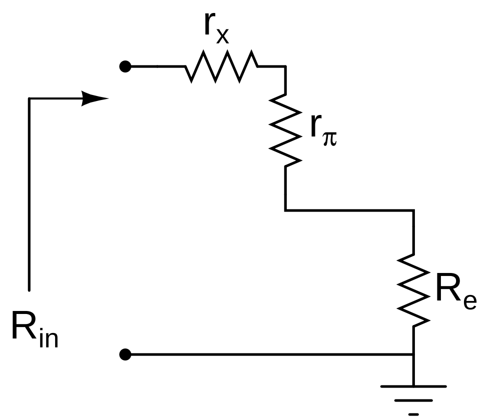

Figure 7: Circuit For Calculating r_x

This circuit schematic shows the input side of the small-signal model. We know R_e, as it is a discrete resistor. We can calculate the value of r_π. We know the overall input resistance ($8.09k\Omega$), so we can solve to find the value of r_x.

$$r_\pi = \frac{V_T}{I_B} = \frac{25mV}{6.66\mu A} = 3.75k\Omega$$

$$R_{in} = r_x + r_\pi + R_e \Rightarrow r_x = r_\pi + R_e - R_{in} = 3.75k\Omega + 6.8k\Omega - 8.08k\Omega = 2.47k\Omega$$

This is a quite reasonable value for r_x, and fits nicely with the rest of our data.

2.6 Cascode Amplifier

The circuit used in sections 2 - 5 was basically a Common-Emitter amplifier. To create an amplifier with more bandwidth and higher gain, we can use a pair of transistors, in a common-emitter/common-base arrangement. This is known as a Cascode amplifier, and has the following schematic.

We design for V_{CC} of 20 v, with I_C of 1 mA. We want to bias the transistors to provide the largest possible output voltage swing, so we bias the base of Q1

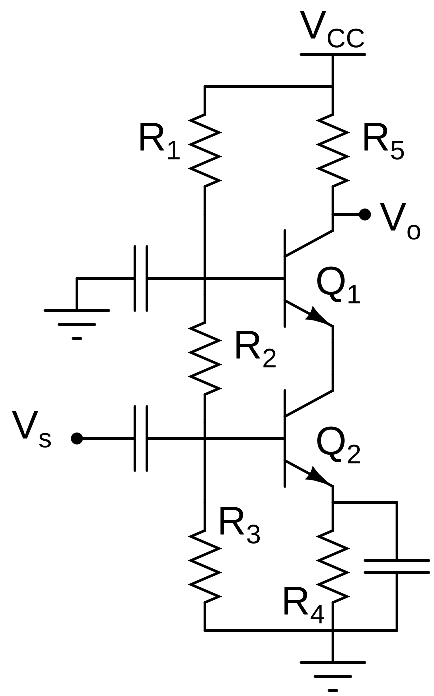

Figure 8: Amplifier Circuit Schematic

to be 10 v, and the base of Q2 to be 5v. We can use R_1, R_2, and R_3 to divide V_{CC}. We want $V_{B1} = 10$ v, so we want $R_1 = R_2 + R_3$. We can pick an arbitrary value for these resistors, so let's have $R_1 = 2k\Omega$ and $R_2 = R_3 = 1k\Omega$.

We let the output voltage DC bias value be 15 v, which will allow the output signal to swing between 10 and 20 volts, or 15 $\pm$ 5 v. There will be a 5v drop across R_5, with 1 mA of current, therefore:

$$R_5 = \frac{5}{1m} = 5k\Omega \approx 4.7k\Omega$$

If the base voltage of Q2 is set to 5v, we know that the emitter voltage for Q2 will be $5 - 0.7 = 4.3v$. Using much the same argument as for R_5, we know both the voltage drop and current through R_4, so we can find its optimal value:

$$R_4 = \frac{4.3}{1m} = 4.3k\Omega \approx 4.7k\Omega$$

We use a $47\mu F$ capacitor for the emitter capacitor, and $10\mu F$ capacitors for the input line coupling capacitors. We measure the voltages at the input and output, and divide accordingly to find the voltage gain. Our results of the frequency sweep are summarized in the table and chart.

Frequency Response of Cascode Amplifier

Frequency (Hz)	Gain (V/V)
10	2.6
100	19.6
1000	67.6
10000	66
20000	74
30000	68
40000	61
50000	61
60000	58
70000	54
80000	49
90000	52.72
100000	39
110000	39
120000	42
130000	37
140000	35
150000	32
160000	31

For this Cascode amplifier, we are anticipating to see a mid-band gain of approximately 150, with a bandwidth in the range of 300 kHz.

We need to calculate the bandwidth of this amplifier. To do so, we measure the upper corner frequency. The maximum gain we see for this amplifier is 67.6, which was lower than expected. This leads to a -3 dB point of 0.707 * 67.6 = 47.79. This occurs at a frequency of only about 100 kHz, much lower than expected. These discrepancies could be caused by more noise in the system. Also, to even get this amplifier to operate properly with little noise, we required a human-body ground on the emitter-side of R_2. If we weren't touching that side of R_2, the circuit wouldn't do anything but generate random noise.

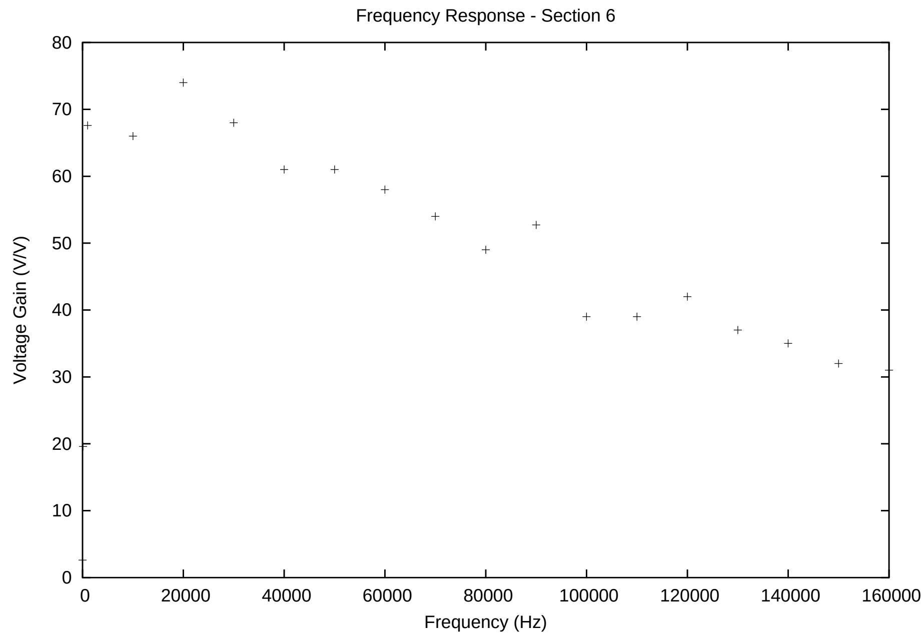

Figure 9: Frequency Response - Section 6

2.7 Common-Collector/Common-Base Transistor Pair

With common collectors and bases, this amplifier should be able to operate with a larger bandwidth, at the expense of some of the gain.

We use $10\mu F$ capacitors for the coupling capacitors, and a $47\mu F$ capacitor between the emitters. With a 1 mA collector current in Q2, we can bias the emitters at 5v, requiring resistors of approximately $4.7k\Omega$. We let the DC bias output voltage be 10v, leaving us with a $\pm 5v$ swing for the output signal.

To find the gain and bandwidth, we make a series of measurements of V_o and V_s, versus frequency. We have tabulated the gain vs. frequency data and created a plot.

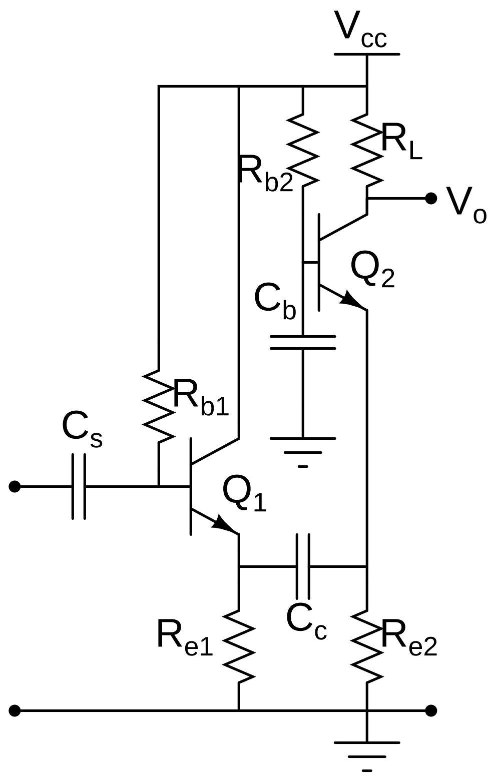

Figure 10: Amplifier Circuit Schematic

Frequency (Hz)	Gain (V/V)
10	0.98
100	52.5
1 k	63.8
10 k	67.1
100 k	71.7
200 k	61.8
300 k	52.7
400 k	42.4
500 k	36.8
600 k	30.9
700 k	28.7
800 k	25.4
900 k	25.4
1 M	19.4

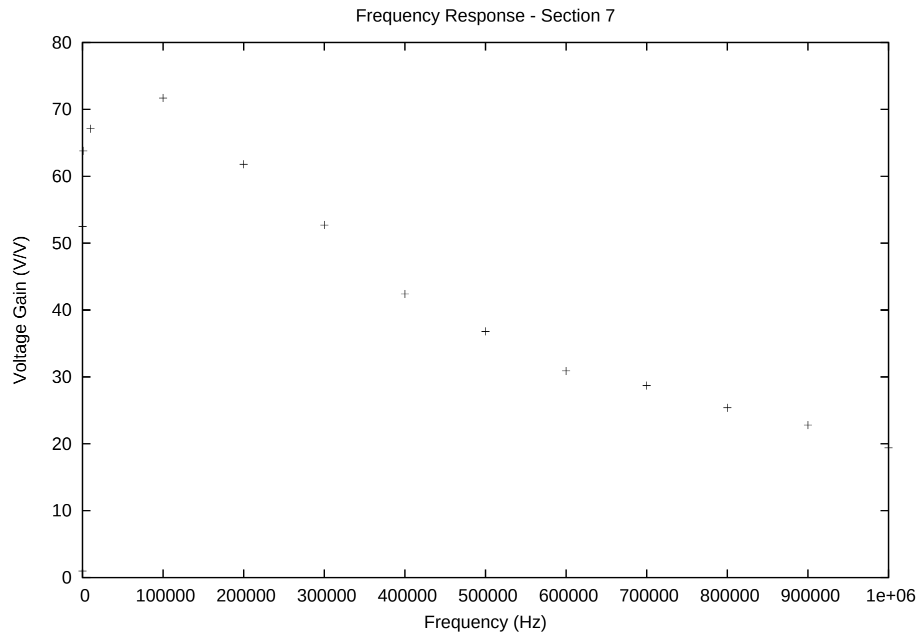

Figure 11: Frequency Response - Section 7

The lower corner frequency is caused by the coupling capacitors shorting out from the very low frequency. This occurs at approximately 90 Hz. The -3 dB point is at 0.707 * 71.7, which is the maximum gain. The -3 db point is 50, which occurs both at 90 Hz and 330 kHz. These are the two corner frequencies, which leave us with a bandwidth of 330 kHz. The mid-band gain is 71.7 V/V. The upper corner frequency is caused by the transistors' high-frequency limitations.

Overall, we had a similar gain, but a larger bandwidth. The max gain was 71.7, with 330 kHz of bandwidth.

2.8 Comprarison of Amplifier Topologies

For this section, we are simply asked to compare the circuits of the previous sections. There were three amplifiers used overall. For sections 1 - 5, we used a basic, single-transistor, current-follower. This showed acceptable gain and bandwidth. Both gain and bandwidth improved when using a pair of transistors in an amplifier, so they may be the best option for a quality amplifier design. The gain and bandwidth both improved from the Cascode amplifier to the CC-CB amplifier, a result not entirely expected. We thought that the CC-CB amplifier would have lower mid-band gain than the Cascode, but our experimental data did not show that.

3 Conclusion

We have certainly learned a great deal about the "Real World" behavior of a BJT. Using the Hybrid-π model, while certainly not the easiest or simplest small-signal model, it fairly accurately modeled the circuit behavior we observed and measured. Noisy signals, even directly out of our function generators, amplified by our simple circuits, were difficult to accurately quantify in the oscilloscope, and certainly contributed to errors and uncertainties in our data and calculations. Part of the problem was the very high gain of these amplifiers, forcing us to use incredibly tiny input signals to avoid output clipping. These very small input signals were difficult to observer clearly on the oscilloscopes, and the percentage noise in the tiny signal is much higher than a larger signal.