Non-ideal Behavior of Electronic Components at High Frequencies and Associated Measurement Problems

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October 27, 2006

Abstract

In the world of electronics, no device is ideal. Inductors possess parasitic resistance and capacitance, and even a simple wire has a small amount of inherent capacitance. Measurement devices, expecially oscilloscope probes and coaxial cables, also exhibit capacitive and resistive effects. In this lab experiment, we investigate these small, normally insignificant properties, that often manifest themselves at high frequencies.

1   Introduction

The oscilloscope is one of the most important tools of an electrical engineer. Like every tool, it is not perfect, and knowing these imperfections is very important to any user. The probes used in conjunction with the oscilloscope possess inherent capacitance. Under normal conditions, these capacitances are inconsequential, but very high frequency signals can be affected by this capacitance. We will investigate the amount and arrangement of the probe capacitance. We will also be investigating the effect of resonance in an inductor, capacitor, resistor circuit, of both the series and parallel configurations. The effect of the quality factor (Q) on the circuits' bandwidth will also be touched on. Lastly, we investigate the parasitic capacitance of a resistor, as well as the parasitic capacitance and resistance of an inductor.

2 Experiments

2.1 Shunt Capacitance of the Oscilloscope Probes

As with most devices, the probes of an oscilloscope have parasitic capacitance. At high frequencies, these capacitances can have a significant effect on the signal. Here, we try to calculate a numeric value for this capacitance using a simple voltage divider. We are recording V_{in} on channel 1, and V_o on channel 2.
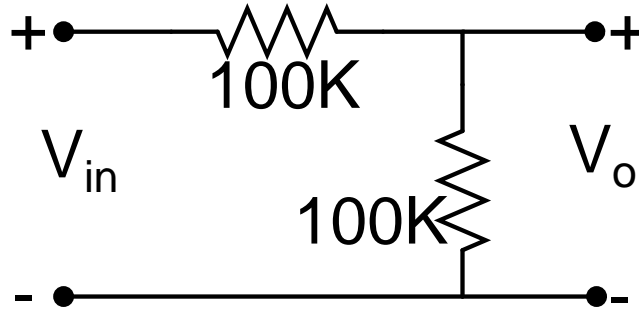

Figure 1: Circuit Schematic - Section 1

We measured both the input and output voltages, which were used to calculate the gain, $A_v = |V_o/V_{in}|$.

Frequency (kHz)	Gain (V/V)
1	0.485
5	0.475
10	0.457
20	0.410
30	0.361
40	0.316
45	0.289
50	0.274
100	0.164
200	0.090
400	0.043
600	0.031
800	0.026
1000	0.021

Frequency Response

A chart of the previous data is useful to observe the behavior of the circuit. As you can see, this response appears to be a low-pass filter.
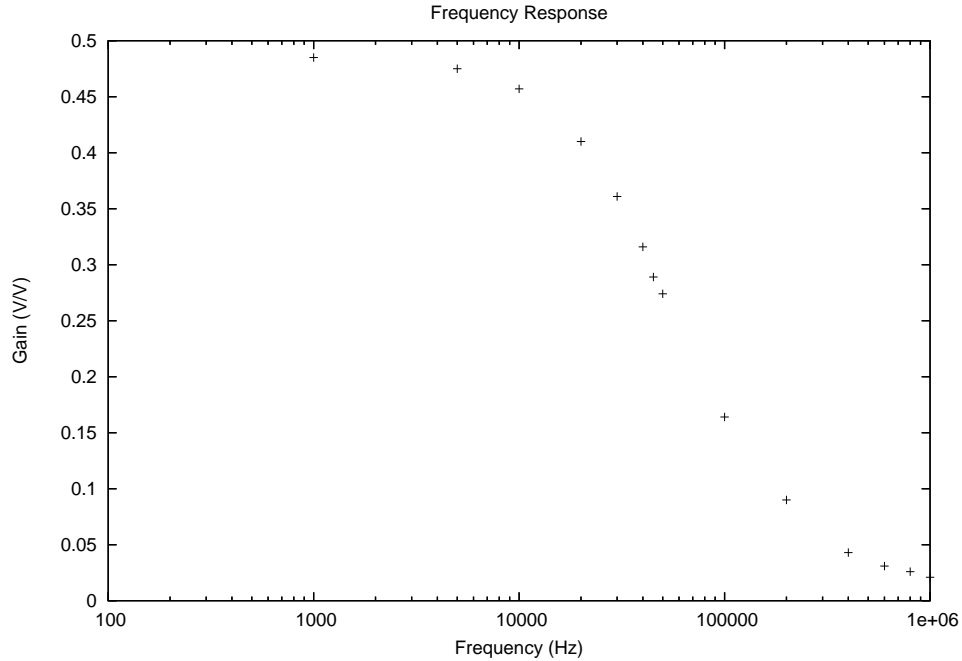
Figure 2: Frequency Response of Voltage Divider

To calculate a value for the shunt capacitance, we can use the concept of a time constant in an RC-circuit. When driven by a square wave, the output voltage lags behind the pulse, exhibiting an exponential response, characterizd by the time-constant. For an RC-circuit, $\tau = RC$. The time-constant defines how much time the output signal requires to reach 63% of the maximum output signal. This number is based on the equation for the step response of the RC-circuit:

$$v(t) = 1 - e^{-t/\tau} = 1 - e^{-t/(RC)}$$

When $t = RC$,

$$v(t) = 1 - e^{-1} = 0.632$$

In figure 3, I have marked both the peak-to-peak voltage, as well as the time-constant's voltage. As you can see, the circuit requires $4.8\mu s$ to reach 63% of the maximum output. For an RC-circuit, $\tau = RC$. We know the value of R, as it is a discrete resistor value, and we have just measured τ, so we can now solve for the unknown capacitance. Since there are two resistances involved in this circuit, the capacitor sees the parallel combination of the two resistances:

$$R_{eq} = 100K\Omega || 100K\Omega = 50K\Omega$$

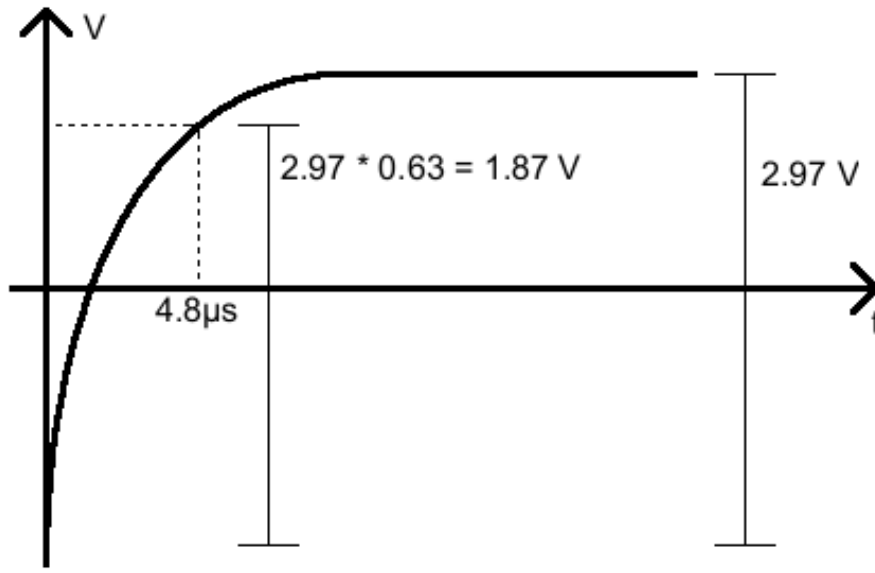$$\tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{4.8\mu s}{50K\Omega} = 96pF$$

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Figure 3: Measuring the Time-Constant

2.2 Series RLC Circuit

Here, we are creating an RLC circuit with a resonant frequency of $2kHz$. Since the lab manual is normally very sparse, and here, incorrect, we are going to use a resistance of 10Ω, not 0Ω as specified. The circuit schematic is in the following figure.

We know that R will have a value of 10Ω. Since we only have one inductor available, with a nominal value of 100 mH, and an actual value of 99.7 mH, this is the inductor we must use. That leaves the capacitor's value left to be found, which we calculate through the specified resonance frequency of 2 KHz:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 f_r{}^2 L} = \frac{1}{4\pi^2 (2000)^2 \cdot 99.7m} = 63.52nF$$

Since I do not have any 63.52 nF capacitors, I must approximate this value by combining the following capacitors, which produces an equivalent capacitance of 60 nF.

Unfortunately, the ceramic disc capacitors we have are 50% tolerance. When I measured the actual equivalent value of my capacitor network, I had a value of 81.5 nF. If we use this value to recalculate the resonant frequency, we get:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{99.7m \cdot 81.5n}} = 1.76kHz$$

The oscilloscopes we have to use can measure the phase difference between the two input signals, which is a very easy and accurate method of measuring the
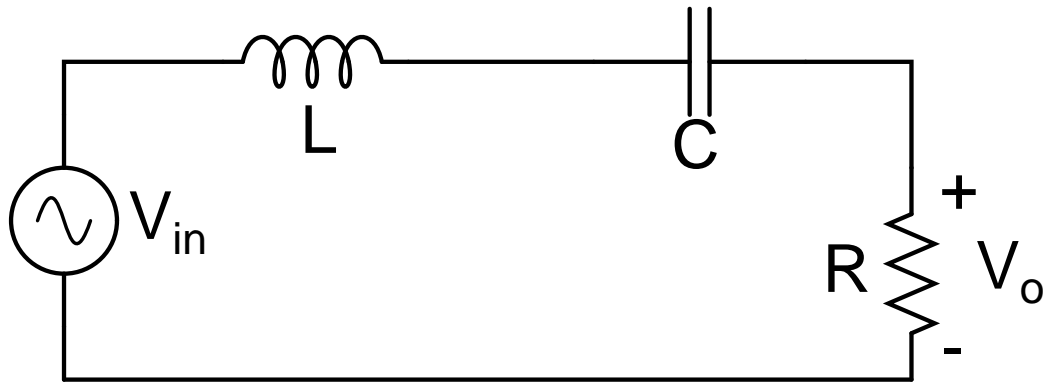
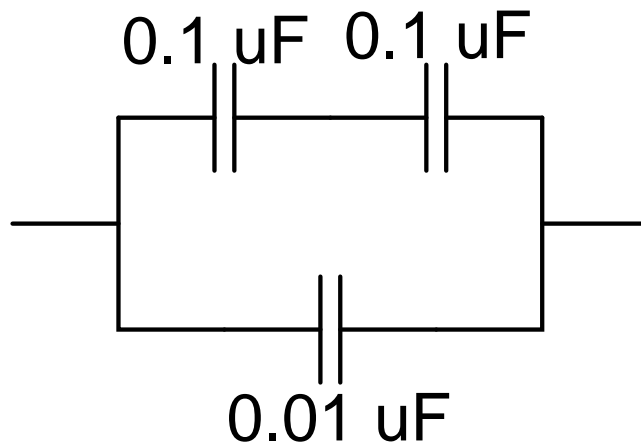Figure 4: Circuit Schematic - Section 2

Figure 5: Circuit Schematic - Section 2

resonant and corner frequencies. When the phase between the input and output signals is zero, we know that we are at the resonant frequency. When the phase is $\pm 45°$, we are at the respective corner frequencies. Using the function generator, we vary the input frequency until we reach one of the three important points. If we were to plot the frequency response of the RLC circuit, it would look something like the following figure, where f_r, f_{c1}, and f_{c2} have been labeled.
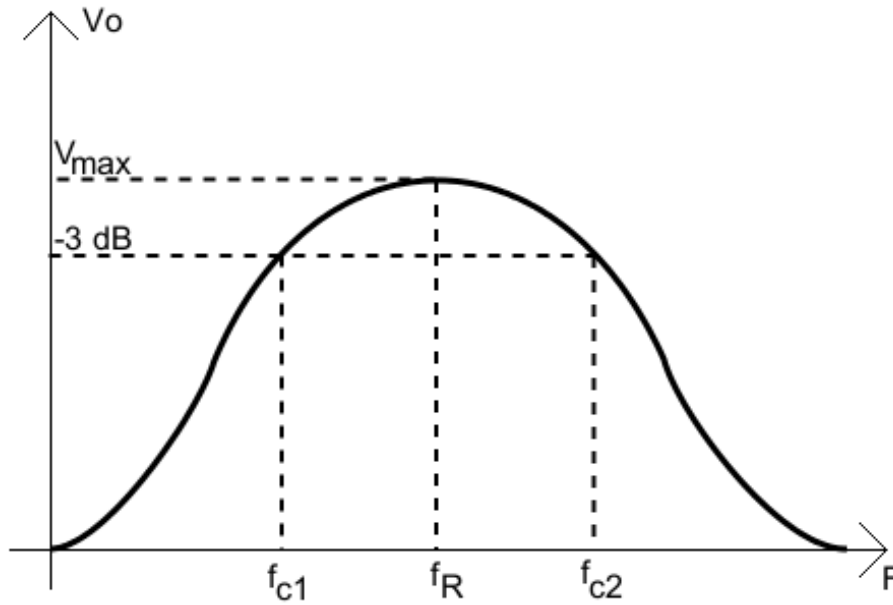
Figure 6: Frequency Response - Section 2

We have measured the following quanities:

Point of Interest	Phase Angle (°)	Frequency (kHz)
Lower Corner Frequency (f_{c1})	$-45°$	1.7
Resonant Frequency (f_r)	$0°$	1.8
Upper Corner Frequency (f_{c2})	$+45°$	2.0

From these values, we can calculate the quality (Q) of this circuit:

$$Q = \frac{f_r}{|f_{c2} - f_{c1}|} = \frac{1800}{260} = 7.13$$

We can calculated an expected value for Q, based on values of the inductance, resistance, and resonant frequency:

$$Q = \frac{2\pi f_r L}{R} = \frac{2\pi \cdot 1800 \cdot 99.7mH}{10\Omega} \approx 112$$

The expected value of Q is nowhere near the calculated value, and we can account for this difference with parasitic resistance. All components involved in this circuit, from the discrete inductor and capacitors, to the breadboard and wires, have a small amount of resistance, which affects the Q value. We can back-substitute the experimentally found Q value into our component-based

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equation, to find the actual equivalent resistance:

$$Q = \frac{2\pi f_r L}{R} \Rightarrow 7.13 = \frac{2\pi \cdot 1800 \cdot 99.7m}{R_{total}} \Rightarrow R_{total} = 158\Omega$$

We can use this resistance with our known resistor value to calculate the parasitic resistance:

$$R_{parasitic} = R_{total} - R_{discrete} = 158\Omega - 10\Omega = 148\Omega$$

We are also interested in calculating the complex impedance $Z_{in}(j\omega_0)$ of this circuit, at the resonant frequency.

$$Z_{eq} = Z_L + Z_C + Z_R = j\omega L + \frac{1}{j\omega C} + R_{total}$$

$$= j \cdot 1800 \cdot 99.7m + -j \cdot \frac{1}{1800 \cdot 81.5n} + 158$$

$$= 186.8 \cdot j - 6550 \cdot j + 158$$

$$= 158 - 6360 \cdot j$$

2.3 Adjustment of Quality Factor

Often, we wish to specify a Q value for an RLC circuit, and adjust the components to reach this specified value. Based on our previous work, we know that the total resistance in an RLC circuit can have a drastic effect on the Q value. Now, we use the component-based formula for Q to calculate the required total resistance to acheive a Q value of 5:

$$Q = \frac{2\pi f_r L}{R} \Rightarrow 5 = \frac{2\pi \cdot 1800 \cdot 99.7m}{R_{total}} \Rightarrow R_{total} = 225\Omega$$

We know that this total resistance is composed of two sources, the discrete resistor placed in the circuit, as well as the parasitic resistance. We have previously calculated the parasitic resistance, and we use that value to determine the required discrete resistance to place in the circuit:

$$R_{new} = R_{total} - Rparasitic = 225\Omega - 148\Omega = 77\Omega$$

We can construct a resistor network consisting of two resistors, one of 30Ω, and one of 47Ω. These will be placed in series to produce a total resistance of 77Ω. Using the oscilloscope as before, measuring the phase to find the locations of the three important points, f_r, f_{c1}, and f_{c2}, we find the following values:

Point of Interest	Phase Angle (°)	Frequency (kHz)
Lower Corner Frequency (f_{c1})	$-45°$	1.73
Resonant Frequency (f_r)	$0°$	1.90
Upper Corner Frequency (f_{c2})	$+45°$	2.14

Using the frequency-based formula for the Q value, we find:

$$Q = \frac{f_r}{|f_{c2} - f_{c1}|} = \frac{1900}{|2140 - 1730|} = 4.63$$

When the circuit is driven with a 2kHz square wave, we get the following signal output:
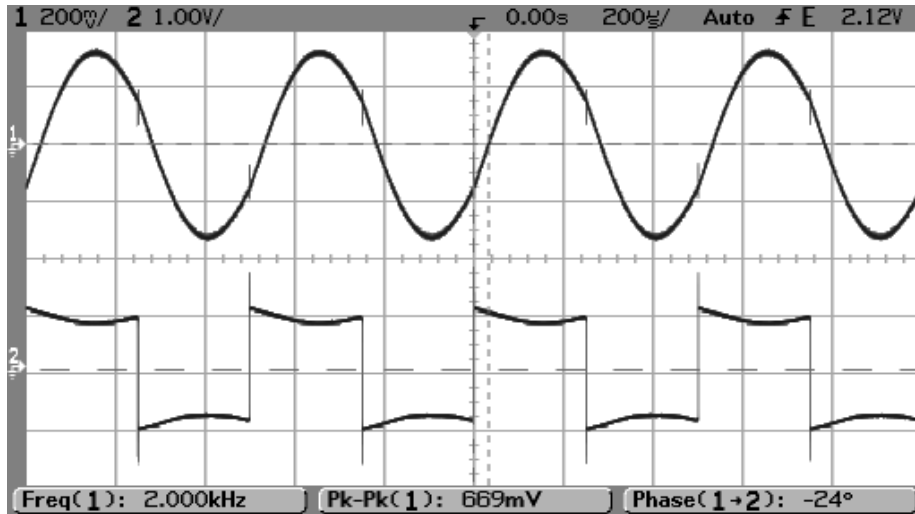

Figure 7: Input and Output Signals

We can use the oscilloscope's Fast Fourier Transform (FFT) feature to visualize the signal spectrum of both the input and output waveforms:

Since the input is a square wave, in the frequency domain, there will be peaks at every odd multiple of the fundamental frequency. These are the odd harmonics of the input signal. For the 2 kHz input signal, we observe peaks in the spectrum at 2k, 6k, 10k, etc.

Since the output is a sine wave, it should have only one component in the frequency domain. In reality, as we can see on the trace, the signal is far from perfect, as other harmonics have a noticable presence. If the sine wave was a pure sine wave, we would have a large spike at the signal's frequency, since the fourier transform of a single sine wave is the impulse function (δ).

2.4 Resonant Frequency with Different Capacitors

Using the other capacitor values available in our laboratory kits, we experimentally find the resonant frequency for capacitances of $0.001\mu F$, $0.01\mu F$, and $0.1\mu F$. As in previous sections, we have used the oscilloscope's phase angle detection ability to simplify and the results are summarized in the following table:
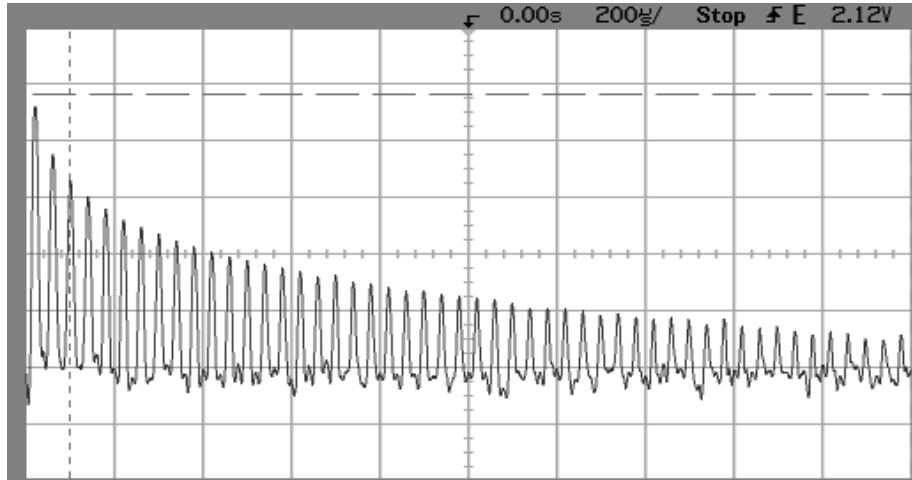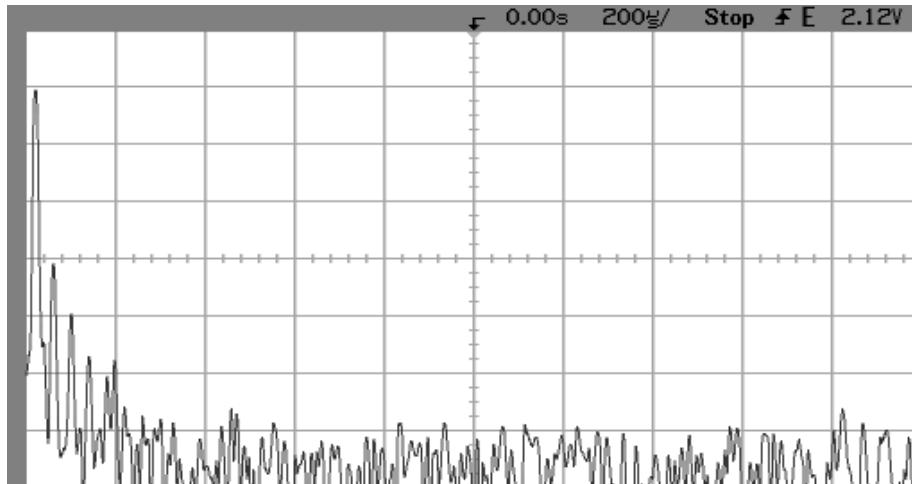
Figure 8: Input Spectrum


Figure 9: Output Spectrum

Nominal Capacitor Value (μF)	Resonant Frequency (kHz)
0.001	15.77
0.01	4.95
0.1	1.53

To find a relationship between f_0 and C, we first look at the formula relating the two variables. It is easiest to see the desired relationship using the following

equation:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \approx \frac{c}{L}$$

This is the equation for a hyperbolic graph, similar to $f(x) = \frac{1}{x}$. If we plot the three data points, we get a simple hyperbola in the first quadrant, asymptotic to both axes. It may be a bit difficult to see with only three data points, but this matches perfectly with the relation found just before

Resonant Frequency versus Capacitance
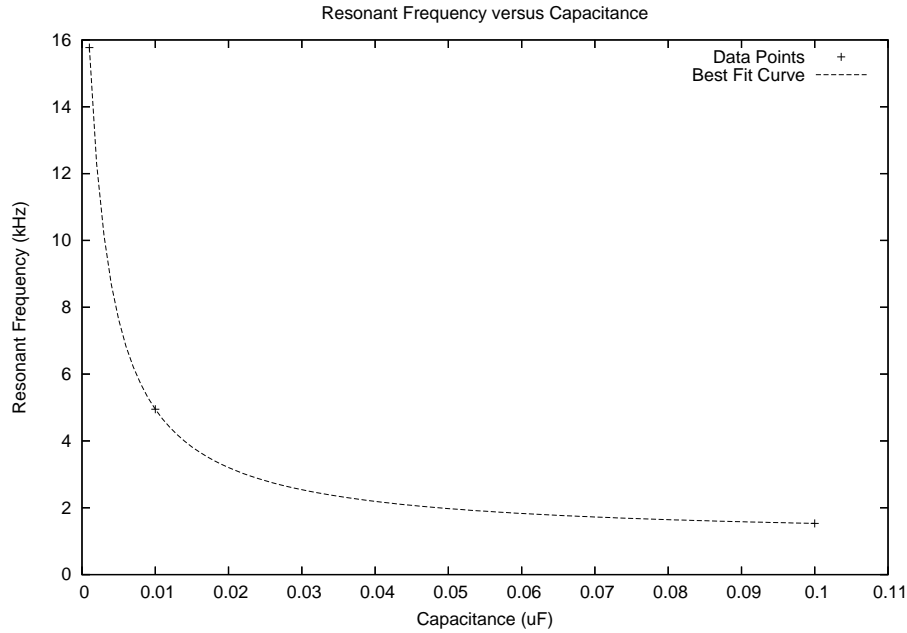
2.5   Parallel RLC Circuit

To create a parallel RLC circuit, we use the same inductor and capacitor network, since the resonant frequency does not depend on the circuit arrangement, only the values of the inductor and capacitor. The schematic for the standard parallel-RLC circuit is included below.

Since we only have access to voltage supples in the lab, we must emulate a current supply with a voltage supply in series with a large resistance. Assuming that the RLC circuit has very little loading effect on the source resistance, this is a fairly good approximation. For the first circuit in this section, we are letting the resistance of the RLC circuit be infinite, effectively creating a short circuit, leaving us with an LC circuit.
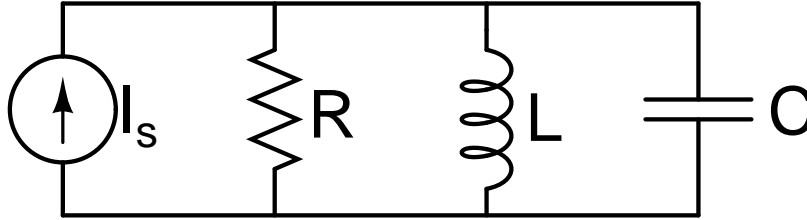
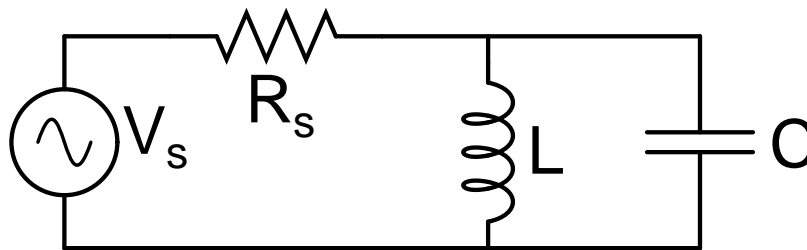Figure 10: Circuit Schematic - Section 5 - Version A

Figure 11: Circuit Schematic - Section 5 - Version B

If we let $R_s = 100$ k Ω, we can calculate the expected Q value:

$$Q = \frac{R_{eff}}{2\pi f_r L} = \frac{100k\Omega}{2\pi \cdot 2000 \cdot 99.7m} = 79.8$$

Using the oscilloscope to measure the phase and frequency of the input and output signals, we find the following points:

Point of Interest	Phase Angle (°)	Frequency (kHz)
Lower Corner Frequency (f_{c1})	$-45°$	1.85
Resonant Frequency (f_r)	$0°$	1.98
Upper Corner Frequency (f_{c2})	$+45°$	2.11

From these values, we can calculate the frequency-based Q:

$$Q = \frac{f_r}{|f_{c2} - f_{c1}|} = \frac{1980}{2110 - 1850} = 7.61$$

The large difference between calculated Q (79.8) and experimental Q (7.61) is again the parasitic resistance present all over this circuit. As in previous

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sections, we want to modify this circuit by including a resistance, with the desired result of a Q value of 5. This new resistance will be placed in parallel with the parasitic resitance. First, we must find what our actual value of equivalent resistance was:

$$Q = 7.61 = \frac{R_{eff1}}{2\pi f_r L} \Rightarrow R_{eff1} = 9.4K\Omega$$

Using the same formula, we can now set $Q = 5.00$, and find our desired value of R_{eff}:

$$Q = 5.00 = \frac{R_{eff2}}{2\pi f_r L} \Rightarrow R_{eff2} = 6.2K\Omega$$

Now, we must resolve the difference between R_{eff1} and R_{eff2}. We do this by placing the new resistance in parallel with the existing circuit:

$$R_{eff} = R_{eff,old}||R_{new} \Rightarrow 6.2K\Omega = 9.4K\Omega||R_{new} \Rightarrow R_{new} = 18.06k\Omega$$

We can use resistors with nominal values of $10K\Omega$, $5.1K\Omega$, and $2.2K\Omega$ to produce an equivalent resistance of $17.3K\Omega$. We place this resistance in parallel with the inductor and capacitor to produce the following circuit.
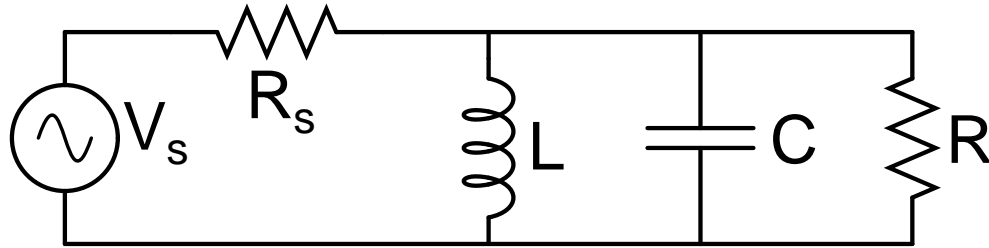

Figure 12: Circuit Schematic - Section 5 - Version C

We measure the resonant and corner frequencies much the same as we have before:

Point of Interest	Phase Angle (°)	Frequency (kHz)
Lower Corner Frequency (f_{c1})	$-45°$	1.787
Resonant Frequency (f_r)	$0°$	1.988
Upper Corner Frequency (f_{c2})	$+45°$	2.188

Using these new frequencies, we calculate the new Q value:

$$Q = \frac{f_r}{|f_{c2} - f_{c1}|} = \frac{1988}{2188 - 1787} = 4.96$$

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To find the admittance of this circuit at the resonant frequency, we first calculate the impedance, and then take the inverse:

$$Z = Z_R||Z_C||Z_L = R_{eff} \left|\left| \frac{1}{j2\pi f_r C} \right|\right| j2\pi f_r L = 6200||-986{\cdot}j||1240{\cdot}j = 2331-3003j$$

We calculate the admittance as $1/Z = 161\mu + 207\mu j$.

2.6 Parasitic Capacitance of Shunt Resistance

Resistors have an inherent capacitance, which may affect high frequency calculations and measurements. We build a simple voltage divider to measure this capacitance, using nominal resistors with values of $R = 1M\Omega$ and $R_L = 5K\Omega$.
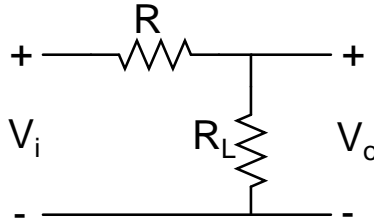

Figure 13: Circuit Schematic - Section 6

To find the capacitance fo this circuit, we want to find the frequency response. We collect data points over a large range of frequencies, and find the -3 dB point, which is located at the corner frequency, f_c. We use this, along with the RC-circuit corner frequency equation to find the value of the capacitance. The data we have collected is reproduced here.

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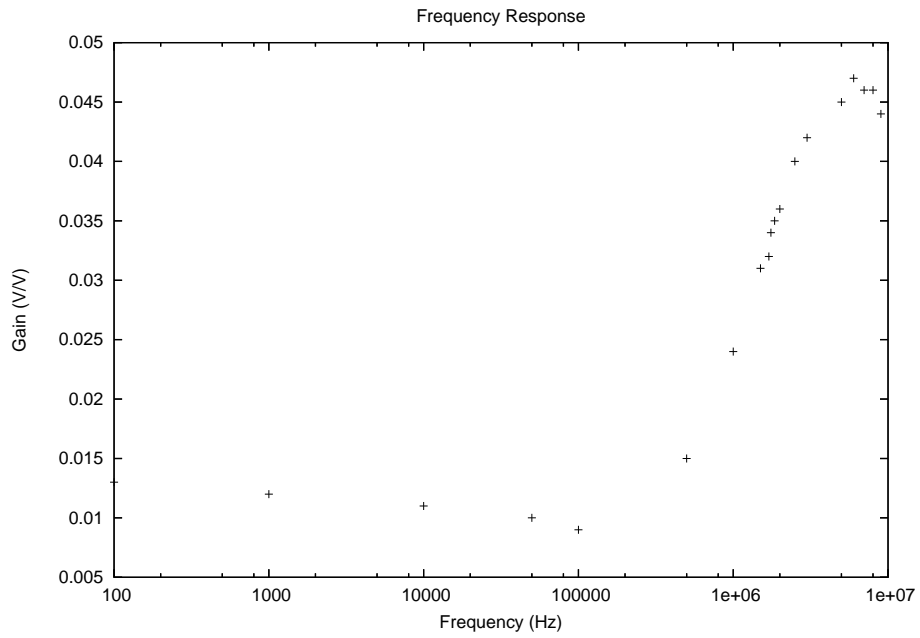| Frequency (Hz)	$V_{in}(V)$	$V_o(V)$	$A_v(V/V)$
100	4.00	0.050	0.013
1000	3.97	0.047	0.012
10K	3.97	0.044	0.011
50K	3.97	0.040	0.010
100K	3.97	0.038	0.009
500K	3.97	0.060	0.015
1M	4.06	0.097	0.024
1.5M	4.06	0.125	0.031
1.7M	4.06	0.131	0.032
1.75M	4.06	0.138	0.034
1.85M	4.06	0.141	0.035
2M	4.13	0.150	0.036
2.5M	4.25	0.170	0.040
3M	4.50	0.190	0.042
5M	5.50	0.250	0.045
6M	6.13	0.290	0.047
7M	6.88	0.315	0.046
8M	7.19	0.330	0.046
9M	6.88	0.300	0.044

Figure 14: Frequency Response

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As you can see, this circuit is behaving like a high-pass filter. From the data points, we calculate that the maximum gain is approximately 0.047. The -3 dB point is therefore $0.047 \cdot 0.707 = 0.033$. The frequency at which the gain is 0.033 is 1.75 mHz, which is the corner frequency of the RC-circuit. We can use the following equation:

$$f_c = \frac{1}{2\pi RC}$$

However, to which resistances and capacitances do the R and C refer to? We must first re-draw the circuit, to include the capacitance of the resistor, as well as the capacitance of the oscilloscope probe.
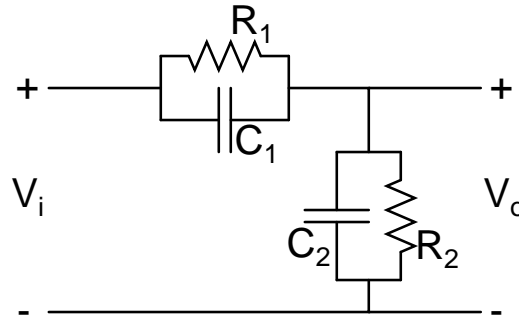

Figure 15: Circuit Schematic - Capacitances Included

Here, R_1 and C_1 are from the $1M\Omega$ resistor. C_2 is the capacitance of the oscilloscope probe. For the small signal analysis, we can combine the resistors in parallel, and combine the capacitors in parallel to get the following schematic:
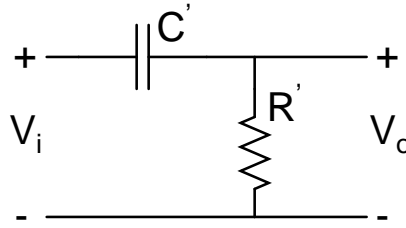

Figure 16: Circuit Schematic - Combined Resistors and Capacitors

The C' and R' are the values used in the previous equation. We know that:

$$R' = R_1 || R_2 = 1M\Omega || 5K\Omega = 4.97K\Omega$$

$$f_c = \frac{1}{2\pi C'R'} \Rightarrow 1.75mHz = \frac{1}{2\pi \cdot 4970 \cdot C'} \Rightarrow C' = 18pF$$

C' is the parallel combination of the resistor's capacitance and the oscilloscope's capacitance. Using the marked value of 15 pF for the probe's capacitance, we find that the capacitance of the shunt resistance is approximately 3 pF. This agrees with our expectations, as the parasitic capacitance of a resistor normally has a very very small effect.

2.7 Parasitic Resitance and Capacitance of an Inductor

A simple inductor, made of a few coils of wire, has a small amount of parasitic resitance and capacitance. We will use the following RL-circuit to investigate the magnitude and properties of these parasitic effects. The value chosen for R is $5.1K\Omega$.
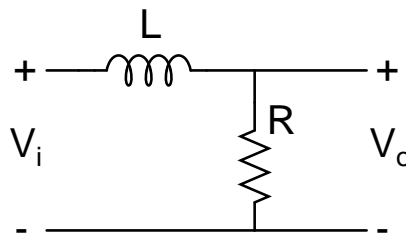

Figure 17: Circuit Schematic - LR-Circuit Divider

Frequency (Hz)	$V_{in}(V)$	$V_o(V)$	$A_v(V/V)$
10	3.94	4.06	1.03
30	4.09	3.90	0.95
100	4.17	4.00	0.96
300	4.18	4.03	0.96
1000	4.19	4.06	0.97
2000	4.31	4.19	0.97
3000	4.44	4.06	0.91
5000	4.44	3.72	0.84
7000	4.50	3.34	0.74
8500	4.50	3.19	0.71
10000	4.44	2.81	0.63
15000	4.47	2.13	0.48
20000	4.48	1.69	0.38
30000	4.44	1.16	0.26
50000	4.44	0.67	0.15
70000	4.44	0.44	0.10
100000	4.44	0.23	0.05
125000	4.42	0.12	0.03
150000	4.42	0.06	0.01
175000	4.40	0.13	0.03
200000	4.46	0.21	0.05
300000	4.44	0.41	0.09
600000	4.44	0.94	0.21
900000	4.50	1.36	0.30
1000000	4.44	1.38	0.31
2000000	4.56	1.97	0.43
3000000	4.94	2.13	0.43
4000000	5.31	2.22	0.42
5000000	5.88	2.56	0.44

There is a commonly used equivalent circuit used to represent the inductor, taking into account the parasitic resistance and capacitance. We substitute this equivalent component into our original circuit:

Using this circuit, our goal is to find the magnitude of the inductor's impedance, as well as the values of L, R_w, and C_w. First, I have prepared a table and plot of frequency versus gain across a wide angle of frequencies.

To start with, we can consider the circuit's behavior at DC, or very low frequency. At DC, inductors become a short circuit, and capacitors become an open circuit. This reduces our circuit down to a simple voltage divider between R_w and R. We measure the voltage in and voltage out to find the value of R_w:

$$V_o = \frac{R}{R_w + R} \cdot V_{in} \Rightarrow \frac{V_o}{V_{in}} = A_v = \frac{5100}{5100 + R_w}$$
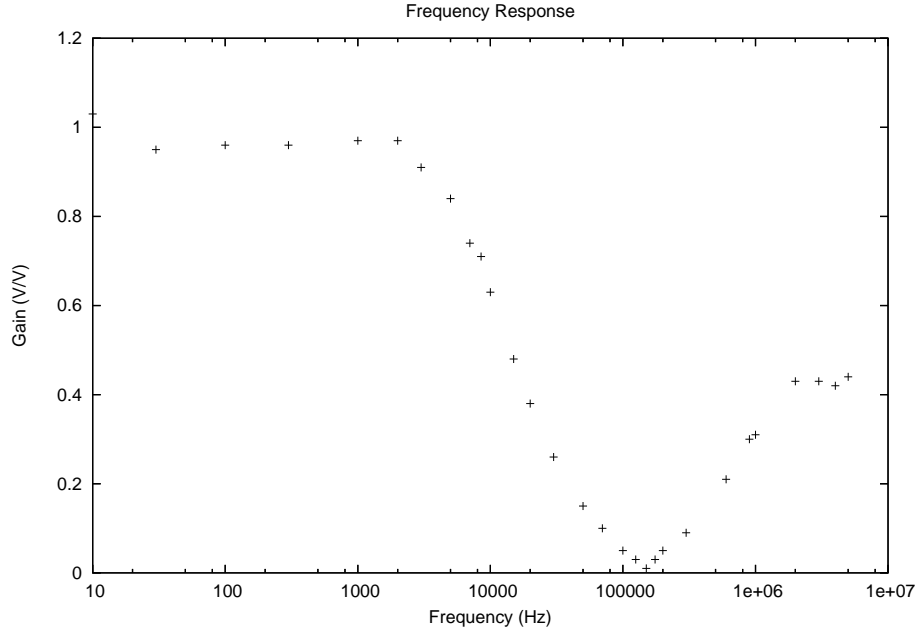
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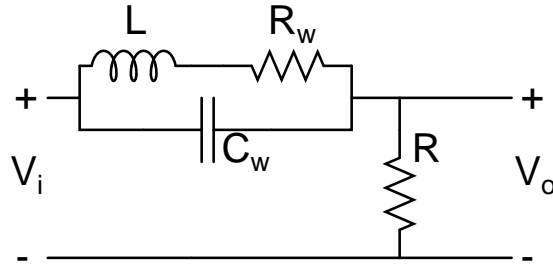Figure 18: Frequency Response of the LR-Circuit in Section 7

Figure 19: Circuit Schematic - LR-Circuit with Equivalent Inductor

$$0.96 = \frac{5100}{5100 + R_w} \Rightarrow R_w = 212.5\Omega$$

We know that the resonant frequency of an RLC-circuit is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$. Using the frequency response data collected for this circuit, we know that the resonant frequency is approximately 150 kHz. Unfortunately, this is not enough information to find both the inductance and capacitance. Fortunately, we can see that as the input frequency is increased past 10^5 Hz, a high-pass filter starts to appear, which is based on the RC-circuit. We can use the fact that the

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gain at the very right side of the frequency response is 0.43 to calculate the -3 dB gain, which we find to be $0.43 \cdot 0.707 = 0.304$, which occurs at approximately 900 kHz. This is the corner frequency of the RC-circuit, enabling us to find the value of C_w:

$$f_c = \frac{1}{2\pi R C_w} \Rightarrow C_w = \frac{1}{2\pi f_C R} = \frac{1}{2\pi \cdot 900000 \cdot (5100 + 212.5)} = 33pF$$

Going back to the resonant frequency equation, we can now find the value of L:

$$f_r = \frac{1}{2\pi \sqrt{LC_w}} \Rightarrow L = \frac{1}{4\pi^2 f_r^2 C} = \frac{1}{4\pi^2 \cdot (150000)^2 \cdot 33pF} = 33.8mH$$

We would expect this value of L to be somewhere around 100 mH, since that is the nominal value. Possible sources of error include discrepancies between the labeled and actual capacitances of the oscilloscope leads, as well as the capacitance inherent in the breadboards.

To find the magnitude of the impedance of the inductor, we return to our original schematic and data tables. Since the impedance is the high-frequency analog of resistance, we simply have a voltage divider circuit. We take our data points, and plot $R \cdot \frac{V_i}{V_o}$. This will give us a chart of impedance. As you can see, the impedance is largest right around the resonant frequency.
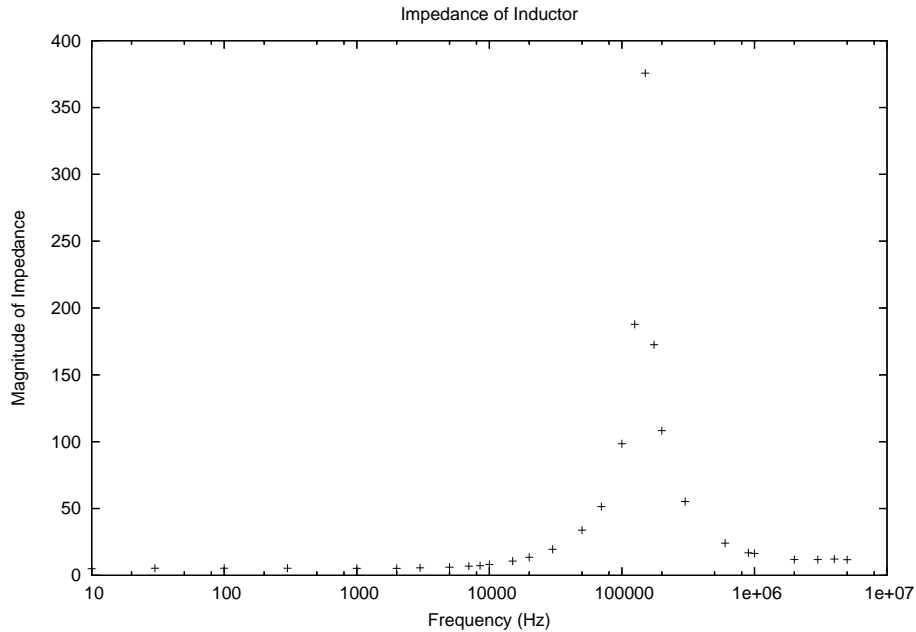

Figure 20: Magnitude of Impedance - Section 7

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3 Results and Conclusion

Altogether, we have investigated both the basic operation of both parallel and series RLC circuits, as well as the effect of non-ideal equipment on high-frequency measurements. Capacitor probes exhibit a parasitic capacitance of approximately 20 pF, while resistors have a smaller capacitance, around 3-5 pF. Inductors also show a small amount of capacitance and resistance, and it is possible to calculate the extent of each. When designing an RLC circuit to meet a certain specification about the Quality factor, we must take care not to forget or disregard the effect of parasitic resistances on the Q value.