Matthew Beckler
Lab Report 1: 9/14 – 10/05
Lab Section Tues Morning

Trajectory Simulations and the Coefficient of Air Drag

Introduction:

When studying projectile motion, it is common to say in the description of a problem, "Disregard the effect of air friction." While this is always convenient to the designers of textbook problems, this does not properly model behavior in reality. In this lab experiment, we decided to take a two-fold approach to the investigation of the effects of air friction on a falling object. Firstly, we investigated whether the academically accepted value of the coefficient of air friction for a smooth plastic ball (ping-pong ball) was consistent with experimental results. Second, through experiment and data analysis, we calculated the coefficient of air friction for a dimpled plastic golf ball. We also compared the findings of those experiments to decide if the results obtained were reasonable when compared to each other.

Predictions:

We predicted that the accepted value would be similar to the results obtained experimentally. When we determined the coefficient of air friction value for the dimpled ball, we thought that it would be around the value for the smooth ball. Later, we reasoned that the value for the dimpled ball should be lower than the value for the smooth ball, because the dimples were added to golf balls exclusively to reduce the air drag associated with the surface.

Description of Experiments:

We used the same general experimental method of gathering data for each ball. We obtained one smooth ping-pong ball and one dimpled golf ball, each about the same mass and size. We decided to use video recorders and computer software to analyze the trajectories of the two objects to determine the positions and times during free-fall. We started out by recording a series of trials in the lobby of Northrop auditorium, filming from one staircase to the other. However, the room proved to be too dark, and it was impossible to view the paths of the balls. We tried again in the stairway leading up to our laboratory room. Even with manual lighting and special settings on the camera, it was still very difficult to observe the balls with any reasonable accuracy. The third time we tried to record video, the conditions were much more favorable, with a bright sun outside to light our experiment. We dropped the smooth ball three times successively, and then dropped the dimpled ball three times. We used specialized software on the computer to mark and quantify the position of the ball at every frame. One of the trials for the smooth ball was unusable because the ball wasn't visible in the video. We used the videos to obtain data relating the position of the ball versus the time after release. This data has been pasted below in the data section. Using the position vs time data, we then extrapolated the velocity and acceleration data using the formulas:

$$\text{Velocity} = \frac{\Delta x}{\Delta t} = \frac{\text{Change in position}}{\text{Change in time}}$$

$$\text{Acceleration} = \frac{\Delta v}{\Delta t} = \frac{\text{Change in velocity}}{\text{Change in time}}$$

This gave us the average velocity and average acceleration over each time interval.

Data:

Ping-Pong Ball:

Experimentally collected data:

Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s2)
3.73	0		
3.8	0.02	0.23	
3.86	0.06	0.68	6.77
3.93	0.17	1.58	13.54
4	0.35	2.71	16.93
4.06	0.51	2.48	-3.39
4.13	0.78	4.06	23.7
4.33	1.7	4.59	2.63
4.4	2.06	5.41	12.41
4.46	2.43	5.63	3.25
4.53	2.84	6.09	6.89

We then created a simulation using the following formula

$$a = g - \left(\frac{1}{2} \cdot \frac{C \cdot \rho \cdot A \cdot v^2}{m}\right)$$

Where:

g = Acceleration due to gravity
C = Coefficient of air friction
ρ = Density of air
A = Frontal area of the object
v = Velocity of the object
m = Mass of the object

When we substituted in for known values, our formula became:

$$a = g - 0.150878 \cdot v^2$$

For our simulation, we used four columns, time, position, velocity, and acceleration. We started our times where the video data started, in order so that the times would align nicely. We decided to use .05 second intervals. The position and velocity started out at zero, and the acceleration started at 9.8, the standard acceleration from gravity. The formulas we used in the spreadsheet are reproduced below, where the following symbols were replaced with the proper cell reference:

A_n	=	Current acceleration
A_{n-1}	=	Previous acceleration
V_n	=	Current velocity
V_{n-1}	=	Previous velocity

Formulas used in simulation:

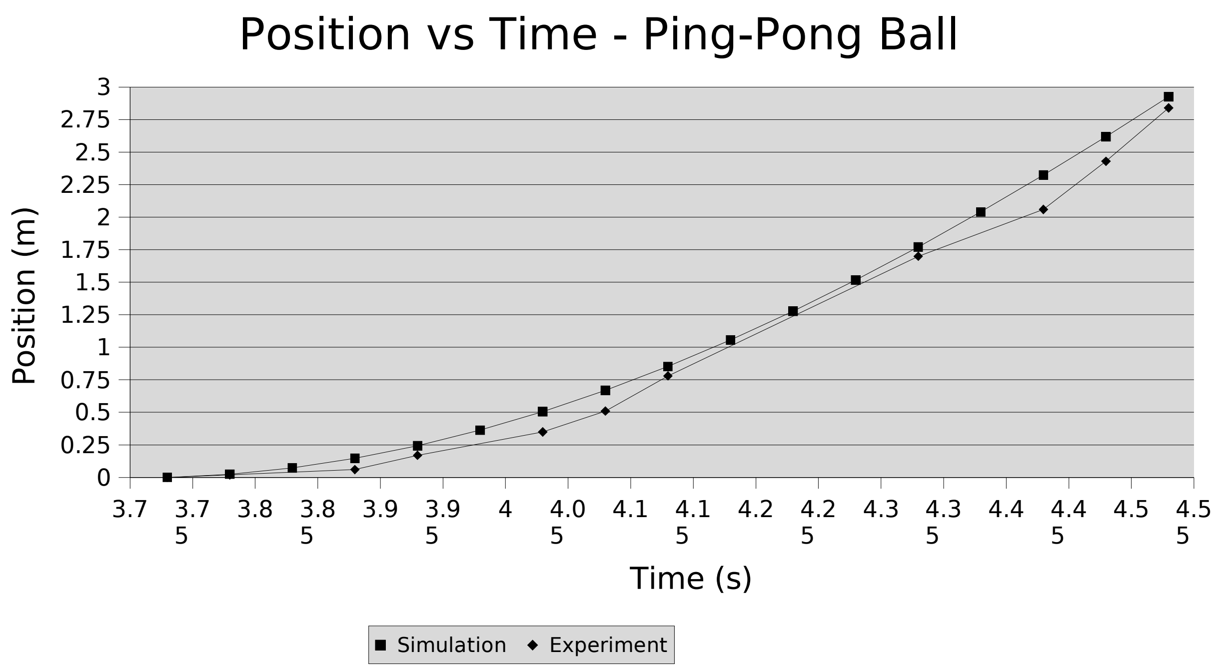
$$A_n = 9.8 - (0.150878 \cdot (V_{n-1})^2)$$

$$V_n = V_{n-1} + A_{n-1} \cdot 0.05$$

We used these formulas with a spreadsheet program to produce the following data:

Time (s)	Position (m)	Velocity (m)	Acceleration (m/s2)
3.73	0	0	9.8
3.78	0.02	0.49	9.76
3.83	0.07	0.98	9.66
3.88	0.15	1.46	9.48
3.93	0.24	1.93	9.24
3.98	0.36	2.4	8.93
4.03	0.51	2.84	8.58
4.08	0.67	3.27	8.18
4.13	0.85	3.68	7.76
4.18	1.06	4.07	7.3
4.23	1.28	4.43	6.83
4.28	1.52	4.78	6.36
4.33	1.77	5.09	5.88
4.38	2.04	5.39	5.42
4.43	2.32	5.66	4.97
4.48	2.62	5.91	4.53
4.53	2.93	6.13	4.12

We created a plot of the position vs time for both the simulation data and the experimental data:



Discussion of Results:

Since we knew the accepted value for the constant C of a smooth ball, we used that value in the simulation to compare to our experimentally determined data. As visible from the chart, the experimental data is quite close to the simulated data. We were very pleased by the correlation shown between the sets of data.

Dimpled Ball:

Experimentally collected data:

Time (s)	Position (m)	Velocity (m)	Acceleration (m/s^2)
17.92	0.01		
17.98	0.01	0	
18.05	0.06	0.65	9.77
18.12	0.13	1.08	6.52
18.18	0.3	2.6	22.81
18.25	0.52	3.25	9.77
18.45	1.32	3.98	3.62
18.52	1.68	5.52	23.22
18.65	2.49	6.03	3.76
18.72	2.85	5.42	-9.02
18.78	3.31	6.94	22.81
18.85	3.74	6.51	-6.52
18.92	4.16	6.29	-3.26

We then created a simulation using the following formula

$$a = g - \left(\frac{1}{2} \cdot \frac{C \cdot \rho \cdot A \cdot v^2}{m}\right)$$

Where:

g = Acceleration due to gravity
C = Coefficient of air friction
ρ = Density of air
A = Frontal area of the object
v = Velocity of the object
m = Mass of the object

Since we did not know the value for C for the dimpled ball, we created a simulation and modified the C value (starting at .5) until the correlation between the data sets became very close. This final value for C was 1.0, and the simulation used the formula:

$$a = g - 0.17677 \cdot v^2$$

For our simulation, we used four columns, time, position, velocity, and acceleration. We started our times where the video data started, in order so that the times would align nicely. We decided to use .05 second intervals. The position and velocity started out at zero, and the acceleration started at 9.8, the standard acceleration from gravity. The formulas we used in the spreadsheet are reproduced below, where the following symbols were replaced with the proper cell reference:

A_n	=	Current acceleration
A_{n-1}	=	Previous acceleration
V_n	=	Current velocity
V_{n-1}	=	Previous velocity

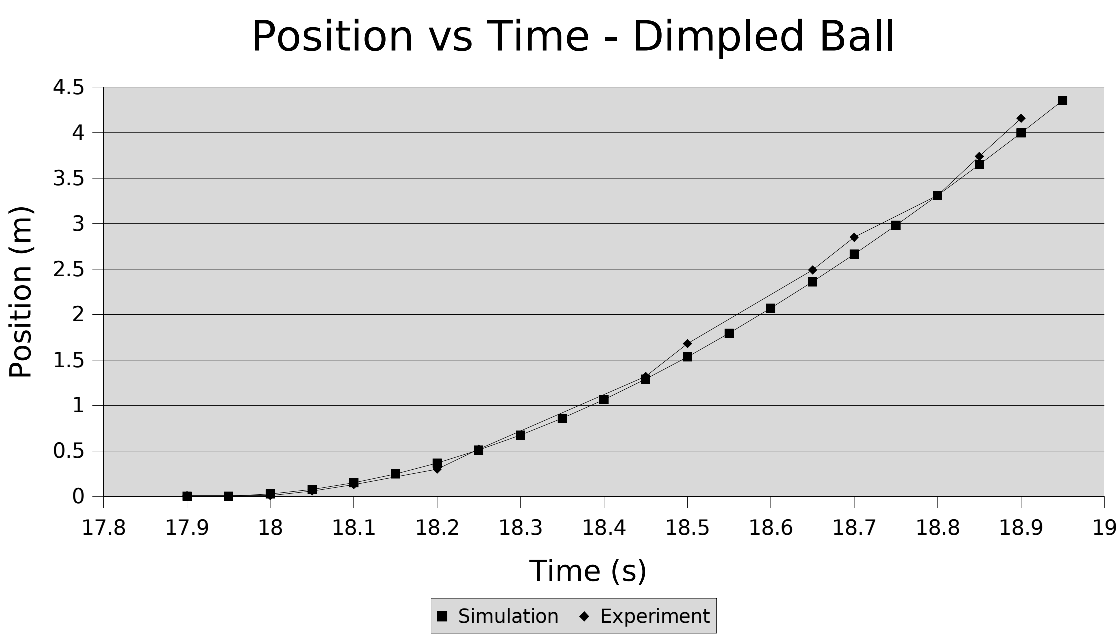
Formulas used in simulation:

$$A_n = 9.8 - (0.17677 \cdot (V_{n-1})^2)$$
$$V_n = V_{n-1} + A_{n-1} \cdot 0.05$$

This produced the following simulation data:

Time (s)	Position (m)	Velocity (m)	Acceleration (m/s²)
17.9	0	0	9.8
17.95	0	0.49	9.77
18	0.03	0.98	9.68
18.05	0.08	1.46	9.53
18.1	0.15	1.94	9.32
18.15	0.25	2.4	9.06
18.2	0.37	2.86	8.75
18.25	0.51	3.3	8.41
18.3	0.67	3.72	8.03
18.35	0.86	4.12	7.63
18.4	1.07	4.5	7.21
18.45	1.29	4.86	6.77
18.5	1.53	5.2	6.34
18.55	1.79	5.51	5.9
18.6	2.07	5.81	5.47
18.65	2.36	6.08	5.06
18.7	2.66	6.34	4.65
18.75	2.98	6.57	4.27
18.8	3.31	6.78	3.9
18.85	3.65	6.98	3.56
18.9	4	7.16	3.24
18.95	4.36	7.32	2.94

We created a plot of the position vs time for both the simulation data and the experimental data:



Results & Analysis:

Ping-Pong Ball:

Based on comparison between the simulation data and the experimental data, we have determined that the accepted value for the coefficient of air friction is accurate. When the data is graphed, the correlation becomes even more obvious.

Dimpled Ball:

After matching the simulation data as close as we could to the experimentally obtained data, the value for C that we got was 1.0, which is a fairly reasonable value, because it is in the general ballpark of normal C-values. If, for example, we had gotten a C-value of 50, that would be unreasonable, the same thing with a C-value of .001. However, as stated in the predictions section, we reasoned that the C-value for the dimpled ball should be less than the C-value for the smooth ball. Since they both ended up to be 1.0, there must have been some inaccuracies in our measurements or analysis.

Error Attribution:

By far the largest problem encountered in the course of this experiment was the use of the low-lighting and low-resolution cameras. It was difficult to see the ball while it was falling, largely due to the fact that the contrast between the white ball and the white stone stairway was very small. Also, when the camera is recording motion, the object gets 'smeared' when traveling too quickly, as it moves quite far in each frame. The software also did not allow for any magnification or zooming of the video file, so obtaining a good degree of clicking accuracy was difficult at the native resolution of the video. For some reason, regardless of whether we calibrated the program to the proper scale, it would not output the position in the correct scale. For example, in our problem, the balls fell a total of approximately four meters. The program returned values between zero and one, which we interpreted to mean the part of our four meter fall. It would have been nice to have been informed beforehand that we would need a reference length visible in the video for calibration of the program.

Estimating Uncertainties:

It is fairly easy to estimate the uncertainty of our measurements from the video footage. We start by estimating the standard mid-range digital video camera's resolution at 320 x 240 pixels. In our experiment we aligned the wide part of the camera with the path of the projectiles. We used approximately 80% of the screen for the path of the balls, so the balls traveled over a distance of about 250 pixels. Since they traveled a real-world distance of about 5 meters, the conversion factor of meters per pixel is:

$$\frac{5(\text{m})}{250(\text{px})} \approx \frac{1(\text{m})}{50(\text{px})} \approx 0.02\left(\frac{\text{m}}{\text{px}}\right) \approx 2\left(\frac{\text{cm}}{\text{px}}\right)$$

Notice that the radii of the ping-pong and dimpled balls were 1.917 cm, and 2.023 cm respectively, and that those values were quite close to the per-pixel error for the video software. With the difficulty of accurately identifying the position of the ball due to reasons outlined in the previous section, if we had missed the 'true' position of the ball by a single pixel, we would have placed the ball a radius away from its actual position.

Conclusion:

While physics professors may ignore forever the effect of air resistance in their examples, we have realized that it isn't that terribly difficult to calculate into your scenarios. The values for C and ρ can be looked up in a reference book, and the values for A, v, and m can be calculated from the object. At that point, the acceleration can be easily incorporated into another problem.